# 433-352 Data on the Web 

Semester 2, 2007<br>Lecturer: Tim Baldwin



## Lecture 7

## Text Categorisation

## What is Text Categorisation?

- Given:

1. a description of a document $x \in X$
2. a fixed set of categories $C=\left\{c_{1}, c_{2}, \ldots, c_{n}\right\}$

- Determine:
the category of $x: c(x) \in C$, where $c(x)$ is a categorisation function whose domain is $X$ and whose range is $C$.
- That is, how do we build categorization functions (classifiers) which can operate over an arbitrary description language and within an arbitrary category space?


## Is This Really Something I Want to Read?

Subject: BUSINESS INVESTMENT
From: COLLINS JAMES [collins55ng@yahoo.co.in](mailto:collins55ng@yahoo.co.in)

Attention: President/Director,

I am the chairman of the contract award committee of the Gold and Natural resources ministry here in Dakar Senegal, for security reasons, I may not wish to disclose the most important thing for now until I hear from you.

After due deliberation with my partner, I decided to forward to you this business proposal, we want you to assist us receive the sum of Twenty eight million, six hundred thousand united state bills (\$28.6M) into your account.
:

## Example Applications of Text Categorisation

- Assign labels to a document, e.g.:
* Yahoo!-style topic label (e.g. sport, news $>$ world $>$ asia $>$ business)
* genre (e.g. job listing, news)
* spam vs. not-spam
* contains-adult-language vs. conforms-to-Bush-sensibilities
- Determine the authorship of a paper
- Document routing
- Indexing (digital libraries, etc.)
- Sorting
- Identifying e-scams
:


## Methods of Categorisation

- Manual categorisation
* very accurate when the job is done by experts
* consistent when the problem size and team are small
* difficult and expensive to scale
- Handcrafted rule-based systems
* e.g., assign a particular category if the document contains a given boolean combination of words
* accuracy is often very high if a rule has been carefully refined over time by a subject expert
* building and maintaining these rules is very expensive
- Automatic classifiers
* $k$-Nearest Neighbours ( $k$-NN)
* Naive Bayes
* support vector machines
* presuppose hand-classified seed data to generalise from
- In practise, commercial systems tend to use bits and pieces of all of these
* Yahoo!, Google Directory (dmoz.com), Reuters, ...


## What are (Supervised) Classifiers?

- Given:

1. a fixed representation language of attributes
2. a fixed set of pre-classified training instances
3. a fixed set of classes $C$
4. a "learner" algorithm which can identify patterns in the training instances

- Estimate:
the category of a novel input $x: c(x) \in C$



## Example Set-up

- Training data:

| Outlook | Temperature | Humidity | Windy | Play |
| :---: | :---: | :---: | :---: | :---: |
| sunny | hot | high | FALSE | no |
| sunny | hot | high | TRUE | no |
| overcast | hot | high | FALSE | yes |
| rainy | mild | high | FALSE | yes |
| rainy | cool | normal | FALSE | yes |
| rainy | cool | normal | TRUE | no |
| overcast | cool | normal | TRUE | yes |

- Test data:

| Outlook | Temperature | Humidity | Windy | Play |
| :---: | :---: | :---: | :---: | :---: |
| sunny | mild | normal | TRUE | $? ? ?$ |

## Supervision

- Supervised methods have prior knowledge of a closed set of classes and instances pre-classified according to those classes, and set out to categorise new instances according to those classes
- Unsupervised methods dynamically discover the classes in the process of categorising the instances [STRONG DEFINITION]

$$
O R
$$

- Unsupervised methods categorise instances without the aid of pre-classified data [WEAK DEFINITION]


## Discussion: supervised or unsupervised?

- Given a set of web documents, identify obvious outliers for manual inspection
- From a set of web documents, filter out the spam documents based on a sample of manually-classified documents
- Classify a set of web documents as belonging to SCC, IN, OUT or OTHER


## Discussion: supervised or unsupervised?

- Given a set of web documents, identify obvious outliers for manual inspection
(strongly) unsupervised
- From a set of web documents, filter out the spam documents based on a sample of manually-classified documents supervised
- Classify a set of web documents as belonging to SCC, IN, OUT or OTHER
(weakly) unsupervised


## ... But First some Basics

- Document representation
- Basics of probability theory
- Basics of entropy


## Document Representation

| id | word | freq |
| :---: | ---: | ---: |
| 1 | a | 233 |
| 2 | aardvark | 0 |
| 3 | aback | 2 |
| 4 | abacus | 0 |


| id | word | freq |
| :---: | :---: | ---: |
| 5 | abalone | 0 |
| 6 | abandon | 1 |
| 7 | abandonment | 0 |
|  | $\vdots$ |  |

$$
\vec{x}=\langle 233,0,2,0,0,1,0, \ldots\rangle
$$

- In practice, we tend to weight each term frequency (see later), and also pre-normalise the document vector to unit length


## (Very) Basics of Probability Theory

- Joint probability $(P(A, B)$ ): the probability of both $A$ and $B$ occurring $=P(A \cap B)$


$$
P(\text { ace }, \text { heart })=\frac{1}{52}, P(\text { heart }, \text { red })=\frac{1}{4}
$$

- Conditional probability $(P(A \mid B))$ : the probability of $A$ occurring given the occurrence of $B=\frac{P(A \cap B)}{P(B)}$


$$
P(\text { ace } \mid \text { heart })=\frac{1}{13}, P(\text { heart } \mid \text { red })=\frac{1}{2}
$$

- Multiplication rule: $P(A \cap B)=P(A \mid B) P(B)=P(B \mid A) P(A)$
- Chain rule: $P\left(A_{1} \cap \ldots \cap A_{n}\right)=P\left(A_{1}\right) P\left(A_{2} \mid A_{1}\right) P\left(A_{3} \mid A_{2} \cap\right.$ $\left.A_{1}\right) \ldots P\left(A_{n} \mid \cap_{i=1}^{n-1} A_{i}\right)$
- Prior probability $(P(A))$ : the probability of $A$ occurring, given no additional knowledge about $A$
- Posterior probability $(P(A \mid B))$ : the probability of $A$ occurring, given background knowledge about event(s) $B$ leading up to $A$
- Independence: $A$ and $B$ are independent iff $P(A \cap B)=$ $P(A) P(B)$


## Binomial Distributions

- A binomial distribution results from a series of independent trials with only two outcomes (i.e. Bernoulli trials)
e.g. multiple coin tosses $(\langle H, T, H, H, \ldots, T\rangle)$
- The probability of an event with probability $p$ occurring exactly $m$ out of $n$ times is given by

$$
P(m, n, p)=\frac{n!}{m!(n-m)!} p^{m}(1-p)^{n-m}
$$

## Binomial Example: $P(m, 10, p=0.1)$



## Multinomial Distributions

- A multinomial distribution results from a series of independent trials with more than two outcomes
e.g. balls in cricket $\left(\left\langle\cdot, \cdot, 1\right.\right.$, out $\left.\left._{\text {LBW }}, \ldots, 4\right\rangle\right)$
- The probability of events $X_{1}, X_{2}, \ldots, X_{n}$ with probabilities $p_{1}, p_{2}, \ldots, p_{n}$ occurring exactly $x_{1}, x_{2}, \ldots, x_{n}$ times, respectively, is given by

$$
P\left(X_{1}=x_{1}, X_{2}=x_{2}, \ldots, X_{n}=x_{n}\right)=\left(\sum_{i} x_{i}\right)!\prod_{i} \frac{p_{i}^{x_{i}}}{x_{i}!}
$$

## Entropy

- Given a probability distribution, the information (in bits) required to predict an event is the distribution's entropy or information value
- The entropy of a discrete random event $x$ with possible states $1, . . n$ is:

$$
\begin{aligned}
H(x) & =-\sum_{i=1}^{n} P(i) \log _{2} P(i) \\
& =\frac{\operatorname{freq}(*) \log _{2}(\text { freq }(*))-\sum_{i=1}^{n} \operatorname{freq}^{(i) \log _{2}(\text { freq }(i))}}{\operatorname{freq}(*)}
\end{aligned}
$$

where $0 \log _{2} 0={ }^{\text {def }} 0$

## Interpreting Entropy Values

- A high entropy value means $x$ is boring (uniform/flat)
- A low entropy value means $x$ is varied ("peaky")


## Entropy of Loaded Dice (1)



## Entropy of Loaded Dice (2)



## Entropy of Loaded Dice (3)



## Entropy of Loaded Dice (4)



## Entropy of Loaded Dice (5)



## Estimating the Probabilities

- The most obvious way of generating the probabilities is via maximum likelihood estimation (MLE), using the frequency counts in the training data:

$$
\begin{aligned}
\hat{P}\left(c_{j}\right) & =\frac{f r e q\left(c_{j}\right)}{\sum_{k} \operatorname{freq}\left(c_{k}\right)} \\
\hat{P}\left(x_{i} \mid c_{j}\right) & =\frac{\operatorname{freq}\left(x_{i}, c_{j}\right)}{\operatorname{freq}\left(c_{j}\right)}
\end{aligned}
$$

- Based on this, our document representation would look something like:

$$
\vec{x}=\langle 0.1,0,0.0002,0,0,0.0001,0, \ldots\rangle
$$

## Modelling Document Similarity: Cosine Similarity

- Given two documents $x$ and $y$, and their corresponding feature vectors $\vec{x}$ and $\vec{y}$, respectively, we can calculate their similarity via their vector cosine:

$$
\operatorname{sim}(x, y)=\frac{\vec{x} \cdot \vec{y}}{|\vec{x}||\vec{y}|}=\frac{\sum_{i} x_{i} y_{i}}{\sqrt{\sum_{i} x_{i}^{2}} \sqrt{\sum_{i} y_{i}^{2}}}
$$

## Cosine Similarity Example

- Calculate the cosine similarity of the following documents:

$$
A=\begin{aligned}
& \text { aardvark back abandon } \\
& \text { abandon abandon }
\end{aligned}
$$

$$
\mathrm{B}=\begin{aligned}
& \text { aardvark abandonment } \\
& \text { back back back }
\end{aligned}
$$

$$
\begin{array}{rlrl}
\vec{A} & =\langle 1,3,0,1\rangle & \vec{B} & =\langle \\
& \equiv\left\langle\frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, 0, \frac{1}{\sqrt{11}}\right\rangle & \equiv\langle \\
\vec{A} \cdot \vec{B} & =\frac{\frac{1}{\sqrt{11}} \times \frac{1}{\sqrt{11}}+\frac{3}{\sqrt{11}} \times 0+0 \times \frac{1}{\sqrt{11}}+\frac{1}{\sqrt{11}} \times \frac{3}{\sqrt{11}}}{1 \times 1} & =\frac{4}{11}
\end{array}
$$

$$
\vec{B}=\langle 1,0,1,3\rangle
$$

$$
\equiv\left\langle\frac{1}{\sqrt{11}}, 0, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}\right\rangle
$$

## Modelling Document Distance: Relative Entropy

- Given two documents $x$ and $y$, and their corresponding feature unit-length vectors $\vec{x}$ and $\vec{y}$, respectively, we can interpret the feature vector as a probability distribution and calculate the relative entropy (or KL divergence):

$$
D(x \| y)=\sum_{i} x_{i}\left(\log _{2} x_{i}-\log _{2} y_{i}\right)
$$

or alternatively skew divergence:

$$
s_{\alpha}(x, y)=D(x \| \alpha y+(1-\alpha) x)
$$

- This causes considerable grief for our MLE-based probabilities: why?
- A simplistic way of getting around this is via Laplacian smoothing:

$$
\begin{array}{r}
\hat{P}\left(c_{j}\right)=\frac{\operatorname{freq}\left(c_{j}\right)+1}{k+\sum_{k} \operatorname{freq}\left(c_{k}\right)} \\
\hat{P}\left(x_{i} \mid c_{j}\right)=\frac{\operatorname{freq}\left(x_{i}, c_{j}\right)+1}{\operatorname{freq}\left(c_{j}\right)+l}
\end{array}
$$

## Relative Entropy Example

- Calculate the relative entropy and skew divergence of the following documents:

```
aardvark back abandon abandon abandon
```

$$
\begin{aligned}
& \text { aardvark abandonment } \\
& \text { back back back }
\end{aligned}
$$

$$
\mathbf{A}=\left\langle\frac{2}{9}, \frac{4}{9}, \frac{1}{9}, \frac{2}{9}\right\rangle \quad \mathbf{B}=\left\langle\frac{2}{9}, \frac{1}{9}, \frac{2}{9}, \frac{4}{9}\right\rangle
$$

$$
D(\mathbf{A} \| \mathbf{B})=\sum_{i} a_{i}\left(\log _{2} a_{i}-\log _{2} b_{i}\right)
$$

$$
\begin{aligned}
&= \frac{2}{9}\left(\log \frac{2}{9}-\log \frac{2}{9}\right)+\frac{4}{9}\left(\log \frac{4}{9}-\log \frac{1}{9}\right)+ \\
& \frac{1}{9}\left(\log \frac{1}{9}-\log \frac{2}{9}\right)+\frac{2}{9}\left(\log \frac{2}{9}-\log \frac{4}{9}\right) \\
& \approx 0.56
\end{aligned}
$$

## Skew Divergence Example

- Calculate the relative entropy and skew divergence of the following documents:
aardvark back abandon
abandon abandon

$$
\mathbf{A}=\langle 0.2,0.6,0.0,0.2\rangle \quad \mathbf{B}=\langle 0.2,0.0,0.2,0.6\rangle
$$

$$
s_{0.99}(\mathbf{A}, \mathbf{B})=D(\mathbf{A} \| 0.99 \mathbf{A}+0.01 \mathbf{B})
$$

$$
=\sum_{i} a_{i}\left(\log a_{i}-\log \left(0.99 b_{i}+0.01 a_{i}\right)\right)
$$

$$
\begin{aligned}
= & 0.2(\log 0.2-\log (0.99 \times 0.2+0.01 \times 0.2))+ \\
& 0.6(\log 0.6-\log (0.99 \times 0.0+0.01 \times 0.6))+ \\
& 0.0(\log 0.0-\log (0.99 \times 0.2+0.01 \times 0.0))+ \\
& 0.2(\log 0.2-\log (0.99 \times 0.6+0.01 \times 0.2)) \\
\approx & 3.67
\end{aligned}
$$

## Nearest Neighbour Classifiers

- There are various ways to combine these document-document scores to form an overall categorisation function, e.g.:
- Method 1: index all training documents, and query the training document set with each test document; classify the test document according to the class of the top-ranked training document [1-NN]
- Method 2: index all training documents, and query the training document set with each test document; classify the test document according to the majority class within the $k$ top-ranked training documents [k-NN]
- Method 3: index all training documents, and query the training document set with each test document; classify the test document according to the class with the best accumulative score [weighted k-NN]
- Method 4: index all training documents, and query the training document set with each test document; classify the test document according to the class with the best accumulative score based on scores, factoring in an offset to indicate the prior expectation of a test document being classified as being a member of that class [offset weighted k-NN]
- Overall advantages of the nearest neighbour approach:
* simple
- Overall disadvantages of the nearest neighbour approach:
* expensive (in terms of index accesses)
$\star$ everything is done at run time (lazy learner)
* prone to bias
* arbitrary $k$ value


## Feature Selection

## Feature Selection

- Classes will tend to have medium-frequency membership, suggesting that we need some extra mechanism of identifying the terms which best discriminate the classes
$\rightarrow$ enter feature selection
- We will focus on greedy inclusion algorithms for feature selection: rank terms in descending order of class discrimination, and select the top $N$ features


## Feature Selection via MI

- Mutual information (MI) provides an information-based estimate of the (in)dependence of two discrete random variables $T$ (term) and $C$ (class):

$$
M I(T, C)=\sum_{t \in\{0,1\}} \sum_{c} P(t, c) \log \frac{P(t, c)}{P(t) P(c)}
$$

- If $T$ and $C$ are independent, $M I(T, C)=0$
- If $T$ and $C$ are positively correlated, $M I(T, C)>0$
- If $T$ and $C$ are negatively correlated, $M I(T, C)<0$
- We select our $N$ "best" features by taking the terms $T$ with the highest MI value
- The method is greedy in that it doesn't take term inter-dependence into consideration
- Bias towards rare uninformative terms
- Example features (on 20 Newsgroups):
sci.electronics: circuit, voltage, amp, ground, copy, battery, electronics, cooling, ... rec.autos: car, cars, engine, ford, dealer, mustang, oil, collision, autos, tires, toyota, ...


## Mutual Information Example

- Perform feature selection over a document collection with the following characteristics:

| Term | Class A | Class B | Total |
| :---: | ---: | ---: | ---: |
| $a$ | 1,000 | 1,000 | 2,000 |
| aardvark | 30 | 50 | 80 |
| aback | 10 | 20 | 30 |
| abacus | 100 | 0 | 10 |
| TOTAL | 1,000 | 1,000 | 2,000 |

$$
\begin{array}{ll}
P(A)=\frac{1000}{2000} & P(B)=\frac{1000}{2000} \\
P(\text { aback })=\frac{30}{2000} & P(\overline{a b a c k})=\frac{1970}{2000} \\
P(\text { aback }, A)=\frac{10}{2000} & P(\overline{a b a c k}, A)=\frac{990}{2000} \\
P(\text { aback } B)=\frac{20}{2000} & P(\overline{a b a c k}, B)=\frac{980}{2000}
\end{array}
$$

$$
\begin{aligned}
M I(\text { aback })= & \frac{990}{2000} \log \frac{\frac{990}{2000}}{\frac{1970000}{200000} 2000}+\frac{980}{2000} \log \frac{\frac{980}{2000}}{\frac{19701000}{2000} \frac{1000}{20}} \\
& +\frac{10}{2000} \log \frac{\frac{10}{2000}}{\frac{30}{2000} \frac{1000}{2000}}+\frac{20}{2000} \log \frac{\frac{20}{2000}}{\frac{30}{2000} \frac{1000}{2000}} \\
= & 0.0009
\end{aligned}
$$

## Feature Selection via $\chi^{2}$

- $\chi^{2}$ ("kai-square") provides an estimate of the level of statistical significance of the correlation between two discrete random variables $T$ (term) and $C$ (class):

|  | Term present | Term absent |
| :---: | :---: | :---: |
| class $=c_{i}$ | $W$ | $X$ |
| class $\neq c_{i}$ | $Y$ | $Z$ |

$$
\chi^{2}=\frac{N(W Z-X Y)^{2}}{(W+X)(W+Y)(X+Z)(Y+Z)}
$$

- The higher the value of $\chi^{2}$, the less confident we are of $T$ and $c_{i}$ being independent
- We select our $N$ "best" features by taking the terms $T$ with the highest $\chi^{2}$ value
- Bias towards frequent uninformative terms


## $\chi^{2}$ Example

- Perform feature selection over a document collection with the following characteristics:

| Term | Class A | Class B | Total |
| :---: | ---: | ---: | ---: |
| $a$ | 1,000 | 1,000 | 2,000 |
| aardvark | 30 | 50 | 80 |
| aback | 10 | 20 | 30 |
| abacus | 100 | 0 | 10 |
| TOTAL | 1,000 | 1,000 | 2,000 |


|  | aback | $\overline{\text { aback }}$ |
| :---: | :---: | :---: |
| $A$ | 10 | 990 |
| $\bar{A}$ | 20 | 980 |

$$
\begin{aligned}
\chi^{2} & =\frac{2000(10 \times 980-990 \times 20)^{2}}{(10+990)(10+20)(990+980)(20+980)} \\
& =3.38
\end{aligned}
$$

## Summary

- What are the basic methods of text categorisation?
- What is the different between supervised and unsupervised classifiers?
- How does the $k$-nearest neighbour method operate, and what are some of the variants on the original algorithm?
-What different methods are used to calculate document similarity?
- How can we perform feature selection?


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